MATHEMATICS SL TZ2

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 41	42 - 53	54 - 65	66 - 77	78 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2009 examination session the IB has produced time zone variants of the Mathematics SL papers.

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

Important information

Teachers are advised that the set of tasks published by the IB for 2009-10 are for those sessions only and should not be used for submission as part of the portfolio after the November 2010 session. They can be used for practice purposes and teachers are encouraged to provide such practice opportunities where time allows. A new set of tasks is in development for 2011-2012. Consult the DP Coordinator Notes and the Online Curriculum Centre (OCC) for updates on the distribution of these tasks. The IB continues to encourage teachers to design and use tasks that fit their own circumstances, providing that these tasks allow the opportunity for students to achieve success at all levels of all criteria.

It is essential for schools to include background information, copies of the tasks, solutions/marking keys and teacher comments with samples submitted. All of these will be very useful in establishing the reasons for the achievement levels awarded by the teacher. It will be even more helpful if teacher expectations on how criteria levels are awarded can be provided in a matrix format, similar to the published overview of assessment criteria, or the matrices in the resources section of the OCC. For schools with more than one teacher for Mathematics SL, such a matrix can serve to help internal standardisation, to ensure consistent marking within the schools.

The range and suitability of the work submitted

This was the first session where teachers were allowed to use the new tasks published for 2009/2010. A 10-mark penalty was applied for using old tasks from the Teacher Support Material documents (TSM). As a result, almost all schools had selected appropriate ones from the published tasks. The most popular Type I task appears to be “Matrix Binomials”, with “Logarithm Bases” the next favourite. For the Type II task, “Body Mass Index” and “Crows Dropping Nuts” were most widely used whereas “Logan’s Logo” was used rarely. Naturally, these would all meet the Internal Assessment requirements without any problems.

School-designed tasks still varied from suitable to unacceptable. Investigations that were overly prescriptive precluded students from achieving higher levels on Criteria C and D. There were also recycled old tasks from the previous course which were not sufficiently modified. Teachers are reminded that before they assign a task to students, they should work through it, to ensure that it matches the assessment criteria well and can allow for candidates to achieve maximum performance. An incomplete portfolio should not be submitted as part of a sample. If selected, it should be accompanied by another portfolio of approximately the same mark.

Candidate performance against each criterion

Criterion A: This remained the easiest for students to attain the highest level of 2. They were nearly all aware of the proper terminology for the required topics and were able to use the corresponding notation consistently. Still, some of them did not recognize the importance of using meaningful variables for different model functions in modelling tasks. Others had repeatedly used calculator/computer notation, like “^”, “*”, [A], which were not penalised by their teachers. The correct use of the “approximately equal” sign should also be enforced, especially in a modelling task, due to the approximate nature of the context.

Criterion B: Most candidates also did well on this criterion and were properly assessed by most teachers. By using appropriate graphs and tables, they would normally achieve levels of 2 or higher. However, a lack of introduction plus poor commentary, a purely “question and answer” format, or student work that requires constant reference to the task statement would lead to a penalty of one mark. The tasks require mathematical writing, not a response to a set of homework exercises.

Type I Criteria C and D: Most students were successful in identifying the patterns and generalizing the results, thus reaching a level 3 in Criterion C, even though in some cases, general statements would appear out of the blue without any analysis or development. A level 4 could then be achieved with a correct and successful mathematical analysis, even if it was not the expected general statement. This could go further up to C5 provided that the general statement was tested against further examples (note the plural!) and/or informally justified. Note that validation of a general statement requires comparing the statement and its results to the actual mathematical behaviour that is the basis of the investigation. Simply substituting new values into a statement and obtaining a result does not verify that the statement reflects the pattern.

Similarly, these students would have no difficulty obtaining a mark of 3 for Criterion D. However, any higher marks proved to be hard to attain since testing for scope and limitations, linked with verification, remained a student weakness. Most students considered only positive integral values, ignoring the possibility of real values for variables. Given the access to technology, it is expected that students will try negatives, fractions, decimals, radicals, etc in their statement as the situation allows. As a result, it was quite common for a D score to be moderated down to a 3 or 4, subject to the presentation of informal justification.

Type II, Criteria C and D: Most candidates did a good job in deriving the suitable models analytically, and then, considering how well the models fit the data, at least, qualitatively. From time to time, some teachers did not regard this to be sufficient for awarding a level 4 in Criterion C, yet a good qualitative response is all that is expected for mathematics SL. Unfortunately, there were still cases in which models were entirely developed by using regression tools, without going through any analytical analysis. This approach allows a maximum mark of C2. To reach the highest level of 5, there must be evidence of applying the student-developed model to further data or another situation. Most candidates also achieved 2 or 3 for Criterion D, indicating an attempt to interpret the reasonableness of the results. However, to be able to make meaningful comments relating to the task or within the context continued to be challenging for them.

Criterion E: The use of technology varied from routine calculations to a full and resourceful application. A lack of background information on available technology makes it difficult to confirm the teacher's assessment. There must be clear evidence on the use of technology within student work to support the highest level of 3 for Criterion E. Quite often teachers' expectations were not consistently applied within the same school samples.

Criterion F: In spite of the fact that F2 was sometimes awarded too casually (even for portfolios with missing parts), Criterion F was generally well understood, with a high level of confirmation. This was mostly due to the fact that the majority of students were given the satisfactory level of 1, as would be expected.

Recommendation and guidance for the teaching of future candidates

Use of tasks

Teachers should feel free to adapt, modify and rewrite the published tasks to best suit their own students due to their diverse background and international nature. They should ensure at the same time that the resulting version provides for full success against the criteria. It would be a good idea to share teacher-designed, non-TSM tasks on the OCC first for advice before actually assigning them to the students.

Specific practice

A variety of tasks, especially for Type II, should be provided. Teachers should develop the process of searching for patterns focussing on the analysis of the data for an investigation task. For the modelling tasks, they should lead students through the development of a mathematical model starting with defining variables, specifying parameters and identifying constraints. Evaluation of results within context is another skill that requires nurturing. Students should also be shown examples of good portfolios, like using correct notation or finding a modelling function algebraically by solving a system of *n* equations with *n* unknowns. Some discussion in class regarding the actual context can help focus the students' interpretations of the scenario.

Comments on the work

Teacher annotations and markings, with precise and specific comments on the students' work serve to facilitate the moderation. They make it easier for the moderators to follow the reasoning behind the teachers' assessment, and also for the students to learn from their own mistakes. Generic feedback such as "good" or "consistent" should be supported with details. Simply rephrasing the criterion level descriptors is not helpful. It is important that all comments are legible. Please take the time to make your comments readable to both the student and the moderator.

Guidance for students

All students should be given copies of a full description of portfolio instructions and the assessment criteria to facilitate their understanding of the entire procedure and setting of their own targets on this internal component. In particular, when a new task is given, they should be clearly informed of the expectations of each assessment criterion.

Technology

Teachers should explore and discuss with students what constitutes resourceful use of technology. Using graphs to reflect or reinforce numerical patterns, using spreadsheets to produce or confirm results for large values of the variable(s), showing an evolution of development of a model through an ever-improving sequence of graphical transformations, or comparing multiple scenarios on the same set of axes to clearly show similarities and differences are some ways in which the effective use of technology can be demonstrated.

External assessment

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 48	49 - 59	60 - 70	71 - 90

The areas of the programme which proved difficult for candidates

The areas of the programme which proved difficult for candidates are:

- logarithms
- trigonometric ratios and angles
- justifying a point of inflexion
- application of expected value
- intersection point of two vectors
- kinematics in context

The levels of knowledge, understanding and skills demonstrated

Candidates in general knew their basic skills and procedures (e.g. finding scalar product, magnitudes of vectors, derivatives with chain and product rules, inverse functions, etc.). But when asked to step beyond the typical formulaic procedure, candidates showed difficulty completing questions that asked to apply a deeper conceptual understanding.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1 (Functions)

Many candidates performed successfully in finding the inverse function, as well as the composite at a specified value of x. Some candidates made arithmetical errors especially if they expanded the binomial before substituting $x = 4$.

Question 2 (Angle between vectors)

Many candidates performed well in finding the magnitudes and scalar product to use the formula for angle between vectors. Some experienced trouble with the arithmetic to obtain the required result. A significant number of candidates isolated the theta answering with $\arccos\left(\frac{-23}{50}\right)$.

Question 3 (Binomial expansion)

The most common error was in (c) where many candidates interpreted the `sixth` term as using $\binom{10}{6}$, with accompanying powers of 4 and 6 in the expression.

Question 4 (Laws of logarithms)

Part (a) proved very accessible, and although many found part (b) accessible as well, a good number of candidates could not complete their way to a final result. Many gave q as a positive value.

Question 5 (Matrix equations)

It was surprising that a good number of candidates did not easily interpret $\boldsymbol{M}$ as the inverse of the inverse. Those who did tended to be successful.

Question 6 (Differentiation)

Many candidates completed parts (a) and (b) successfully. A rare few earned any marks in part (c) - most justifying the point of inflexion with the zero answers in part (b), not thinking that there is more to consider.

Question 7 (Trigonometric equations

Those who realized e^{2x} was a common factor usually earned the first four marks. Few could reason with the given information to solve the equation from there. There were many candidates who attempted some fruitless algebra that did not include factorisation.

Question 8 (Product rule)

A good number of candidates found the correct derivative expressions in (a). Many applied the product rule, although with mixed success. Often the substitution of $\frac{\pi}{3}$ was incomplete or not done at all.

Question 9 (Probability and distributions)

Most candidates completed parts (a), (b) and (c) successfully. Many found the expected value correctly, while some showed difficulty with the arithmetic. Part (e) was often left blank or only superficially attempted. Some found the expected value $\frac{50}{9}$ but did not answer the question about the amount of money.

Question 10 (Vectors)

Very few candidates gave a correct direction vector parallel to the z-axis. Provided they wrote down an equation here they were able to earn most of subsequent marks on follow through. For (b), many found the correct parameter but neglected to confirm it in the other two equations. In (c) some performed a trial and error approach to obtaining an integer parameter and thus did not `show` the mathematical origin of the result. Finding vector $\overrightarrow{AB}$ proved accessible, and a good number of candidates had an appropriate approach to (d), although surprisingly many subtracted $\overrightarrow{OC}$ from $\overrightarrow{AB}$ in finding $\overrightarrow{OD}$.

Question 11 (Kinematics)

Part (a) proved accessible for most. Part (b), simple as it is, proved elusive as many candidates did not make the connection that $v = 0$ when the train stops. Instead, many attempted to find the value of t using $a = \frac{8}{5}$. Few were then successful in part (c).

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 10	11 - 20	21 - 35	36 - 46	47 - 56	57 - 67	68 - 90

The areas of the programme which proved difficult for candidates

The areas of the programme which proved difficult for candidates are:

- Sigma notation and explaining why a geometric series diverges.
- Providing evidence to support a conclusion and expressing clear mathematical reasoning.
- Giving descriptions of transformations.
- Correct use of the discriminant.
- Normal distribution (especially standardization of variables).
- Probability in context; binomial probability.
- Knowing when to use a GDC in a question and how to display evidence of method used when so doing.
- Drawing neat, clear sketches.
- Premature rounding in multiple step questions

The levels of knowledge, understanding and skill demonstrated

- There was a wide range of knowledge, understanding and skills demonstrated with almost all candidates finding questions or parts of questions where they could be successful.
- Many candidates demonstrated skill in using the GDC to find intercepts, binomial probabilities, and definite integrals.
- The majority of candidates showed a good level of knowledge of trigonometry in non-right angled triangles using sine and cosine rules.
- Candidates demonstrated good understanding of box and whisker plot.
- Finding the equation of the normal to a curve was a strength.
- Their work was generally set out in a clear way and this year the vast majority followed the instructions to present their work correctly (Section A on the question paper and Section B on lined paper attached to the back of the question booklet).

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1 (Box-and-whisker plot)

Most candidates were able to find the values for the median, lower quartile, and point b. A large majority answered this question correctly.

Question 2 (Transformations of graphs)

This question was reasonably solved by many students, though a good number confused $f(-x)$ with $-f(x)$ in part (a), thus reflecting the original diagram in the x–axis. Candidates need more practice in correctly and fully describing transformations. There was often confusion between the description of the transformation and the equation that represents it. A fairly low percentage of the candidates used the term ‘translation’.

Question 3 (Solving equations)

Although many students started with an analytical approach, many also realized they were not going further and successfully used their GDC to find the intercepts with the x-axis if they had set the equation equal to 0, or in other cases, they found the intersection of the two graphs. The better candidates drew a reasonable sketch and found the two values without difficulty. A good number of candidates did not provide a sketch, however, and they had more trouble earning the mark for showing method. Accuracy penalties were relatively common on this question.

Question 4 (Triangle trigonometry)

This question was generally well done. Even the weakest candidates often earned marks. Only a very few candidates used a right-angled triangle approach. Almost no candidates realized there was an ambiguous case of the sine rule in part (b). Those who did not lose the mark for accuracy in the previous question often lost it here.

Question 5 (Sigma notation)

This question proved difficult for many candidates. A number of students seemed unfamiliar with sigma notation. Many were successful with part (a), although some listed terms or found an overall sum with no working. The results for part (b) were much more varied. Many candidates did not realize that n was 27 and used 30 instead. Very few candidates gave a complete explanation why the infinite series could not be evaluated; candidates often claimed that the value could not be found because there were an infinite number of terms.

Question 6 (Gradient and normal)

Although the command term "write down" was used in part (a), many candidates still opted for an analytic method for finding the derivative value. Although this value was often incorrect, many candidates knew how to find the equation of the normal and earned follow through marks in part (b).

Question 7 (Quadratic equations)

Although some candidates correctly considered the discriminant to find the possible values of k, many of them did not set it equal to 0, writing an inequality instead. In part (b), some students realized that the discriminants in parts (a) and (b) were the same, earning follow through marks just by writing the same (often incorrect) answers they got in part (a). Many, however, did not see the connection between the two parts.

Question 8 (Areas and volumes)

Many candidates set up a completely correct equation for the area enclosed by the x-axis and the curve. Also, many of them tried an analytic approach which sometimes returned incorrect answers. Using the wrong limits 0 and 6 was a common error.

The formula for the volume of revolution given in the data booklet was seen many times in part (b). Some candidates wrote the integrand incorrectly, either missing the π or not squaring. A good number of students could write a completely correct integral expression for the volume of revolution but fewer could evaluate it correctly as many started an analytical approach instead of using their GDC.

Many candidates did not use a GDC at all in this question. Pages of calculations were produced in an effort to find the area and the volume of revolution. This probably caused a shortage of time for later questions.

Question 9 (Normal distribution and binomial probability)

A significant number of students clearly understood what was asked in part (a) and used the GDC to find the result. However, in part (b), many candidates set the standardized formula equal to the probability (0.85), instead of using the corresponding z-score. Other candidates used the solver on their GDC with the inverse norm function.

A common incorrect approach in part (c) was to attempt to use the means and standard deviations for justification, although many candidates successfully considered probabilities.

A pleasing number of candidates recognized the binomial probability and made progress on part (d).

Question 10 (Trigonometric functions)

Some graphs in part (a) were almost too detailed for just a sketch but more often, the important features were far from clear. Some graphs lacked scales on the axes.

A number of candidates had difficulty finding the period in part (b)(ii) and writing the correct value of q in part (c).

The most common approach in part (d) was to differentiate and set $f'(x) = 0$. Fewer students found the values of x given by the maximum or minimum values on their graphs.

Part (e) proved challenging for many candidates, although if candidates answered this part, they generally did so correctly.

In part (f), many candidates were able to get as far as equating the two derivatives but fewer used their GDC to solve the resulting equation. Again, many had trouble demonstrating their method of solution.

The type of assistance and guidance the teachers should provide for future candidates

Focus on conceptual understanding throughout the programme. Three examples from paper one: most students erroneously think a point of inflexion is where the second derivative is zero. Few understand it as the point at which concavity changes, and so do not consider that the justification of such a point also requires a look left and right of the point in question. Another example is found in understanding that an inverse of an inverse is a return to an original, such as with the matrices of Q5. A third example is found in realising that when an object in motion stops, the velocity is zero at this time.

Candidates should be instructed to consider both geometric and analytical approaches to solving problems to facilitate understanding. Many candidates have difficulty with the underlying concepts as they are only aware of which buttons to press on their calculators. As such, they are unable to apply their knowledge to a problem presented in a slightly different way. When preparing candidates for future examinations, emphasizing a graphical understanding in conjunction with analytical techniques is essential.

It is noticeable that successful candidates have work that is clearly set out while unsuccessful candidates usually display an incoherent structure to their work. An inability to organize one's mathematical thoughts is a drag force in a timed examination. Emphasizing high quality mathematical communication can help students learn to organize their thoughts and become more efficient mathematical thinkers.

Students continue to struggle with the expectations of a "show that" question. Thinking backwards can sometimes be a helpful mental strategy, but the written work must show a deductive path from some mathematical principle that clearly leads to the desired result, without any backwards thinking from the answer given.

Teachers should continue to advise candidates to show their working, particularly when answering a "show that" question or giving a GDC solution. Candidates need to consider that they are expressing their solutions for an examiner who does not know them.

On the GDC paper, candidates need to determine when to use their GDC. There are students who seem not to have had the chance to reflect about this, to discuss it in class. As a result, they do not use their GDCs as they should. They do more analytical work than is expected. On this paper, the GDC can assist with integration, gradient, probability distributions, curve sketching, and solving equations.

Teachers need to ensure that all areas of the syllabus are given adequate treatment.

Candidates need to be aware that premature rounding of answers in intermediate steps can lead to inaccuracy in the final answer.

Teachers should use the “command terms” and stress the nuances of each with their students.